MECHANICS.

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SOLUTIONS TO PROBLEMS.

3\. Proposed by CHARLES E. MYERS, Canton, Ohio.

A spherical air-bubble, having risen from a depth of 1,500 feet in water, was one inch in diameter when it reached the surface; what was its diameter at the point of starting?

I. Solution by the Proposer.

Let x=radius at the point of starting. At the surface the pressure per unit is equal to a column of water 34 feet high, and at a depth of 1500 feet by 1500+34, and since the volumes are inversely as the pressures, we have,

$\frac{4}{3}\pi\left(\frac{1}{2}\right)^3 : \frac{4}{3}\pi x^3 :: 1500+34:34$, or, $6136x^3=17$, whence, $x=.14$ inches and $2x=.28$ inches, the required diameter.

II. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let p=pressure per square inch on the bubble as it emerges at the surface, and p' the pressure per inch on the bubble at the bottom of the 1500 feet of water;then if the temperature does not change and assuming that Boyle's law holds good, namely, that the volume of a given mass of gas, at a constant temperature, is inversely as the pressure, the required diameter$=(p\div p')$.

But both p and p' are influenced by temperature, by atmospheric pressure, which is always changing, by the distance of the place of observation from the earth's centre, which affects the weight of both air and water.

Also the compressibility of the water affects the amount in a column of 1500 feet; and if sea-water is considered, another important factor enters into the calculation as sea-water is heavier and less compressible than pure water.

Let us assume the truth of Boyle's law and also of Charles's law, that the product of the volume and pressure of any gas is proportional to the absolute temperature,calculated from $-460°$F. Assuming also that the temperature of the water is constant throughout the course of the bubble, and, that the weight of a body above the surface of the earth is inversely as the square of its distance from the centre, but below the surface directly as the distance; that the compressibility of pure water is .000048 per atmosphere of 14.7 lbs. to the square inch; that the compressibility of sea-water is $.925\times.000048=.000044$ per atmosphere and its specific gravity is 1.0263; we may proceed as follows:

Let r=the distance of the centre of the earth from the surface of the water; t=temperature of the bubble at bottom, t'=the temperature of it at the top; h=height in feet of a column of water of one-inch cross-section and weighing 14.7 lbs; h'=height of a column of water rising from the neighborhood of the emerged bubble,and supported exactly by the atmospheric pressure at the particular time and place. Then, approximately, the standard atmospheric pressure is to the pressure p as

$$\int_0^h \frac{r^2}{(r+h-x)^2} \cdot \frac{h^2\,dx}{h^2-.000048\int_0^x \frac{r^2\,dy}{(r+h-y)^2}} : \int_0^{h'} \frac{r^2}{(r+h'-x)^2} \cdot$$

$$\frac{h^2\,dx}{h^2-.000048\int_0^x \frac{r^2\,dy}{(r+h'-y)^2}}$$

for pure water and the same formula for sea water by putting .000044 for .000048 with different values for h and h'. However we will use the ordinary formula $h:h'=14.7:p$, or $p=\frac{14.7h'}{h}$.

We next have $p:p'=h':k$, where

$$k=h'+\int_0^{1500} \frac{r-x}{r} \cdot \frac{h^2\,dx}{h^2-.000048\left(h'+\int_0^x \frac{r-y}{r}\,dy\right)}$$

$$=h'+\int_0^{1500} \frac{2h^2(r-x)dx}{2r(h^2-000048h')-.000096rx+.000048x^2},$$

$$=\frac{h'}{.000048}\left[\log 2r(h^2-.000048h')-\log\left\{2r(h^2-.000048h')-.144r+108\right\}\right],$$

for pure water,

$$\frac{h'}{.000044}\left[\log 2r(h_1^2-.000044h_1')-\log\left\{2r(h_1^2-.000044h_1')-.132r+99\right\}\right]$$

for sea water.

The semi-axes of the earth are $a=20926202$ feet, $c=20854895$ feet and we obtain r for any latitude θ from the formula $r=\frac{ac}{(a^2\sin^2\theta+c^2\cos^2\theta)^{\frac{1}{2}}}$ for Greenwich $\theta=51^\circ 21' 38\frac{1}{2}''$ and $r=20882610$. If the surface of the water in question be any where in this latitude and at sea-level, let $t'=(460+60)^\circ$F, $t=(460+75)^\circ$F; also let $h=33.35$, $h'=32.90$ for fresh water; then for sea-water $h_1=32.50$ $h_1'=32.057$; hence $p=14.502$ lbs. and $k=32.90+23171302$ (log 46452159574$-$log 46449152586)$=1532.9502$ for pure water, $k'=32.06+24005682$ (log 44107504286$-$log 44104747880)$=1531.6325$ for sea water.

Now from $p:p'=h':k$, $k'=675.679$ lbs. for pure water.

$p/p'=.0214629$. $p'=692.882$ lbs. for sea water.

$p/p'=.0209297$. $\frac{t}{t'}=\frac{535}{520}=1.028846$

$\therefore$ for fresh water, diameter$=\{(.0214629)(1.028846)\}^{\frac{1}{3}}=.28055161$;
for sea-water diameter$=\{(.02092997)(1.028846)\}^{\frac{1}{3}}=.27817051$.

If the temperature does not change then
for pure water the diameter $=(.0214629)^{\frac{1}{3}}=.27790474$,
for sea-water the diameter$=(.02092997)^{\frac{1}{3}}=.27554582$.